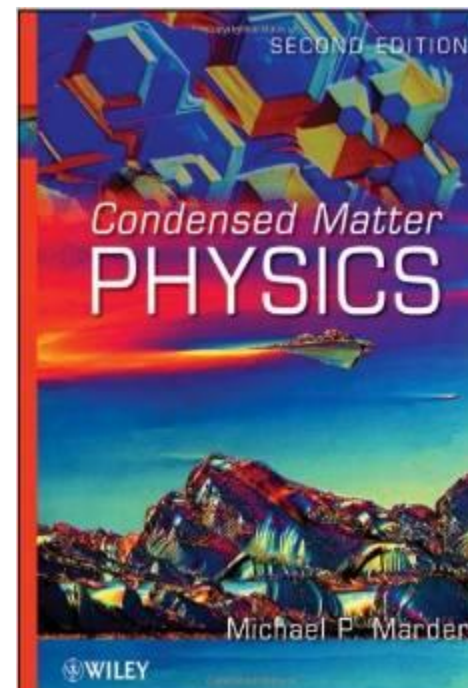
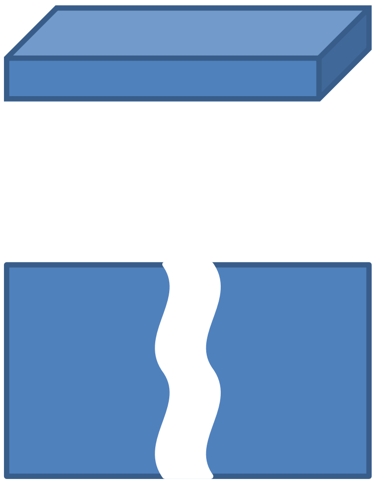
Magnetism to Spintronics

Introduction to Solid State Physics
Kittel 8th ed
Chap. 11-13,

Condensed Matter Physics
Marder 2nd ed
Chap. 24-26,



為什麼(大部分)磁鐵打破後會相斥?



Cooperative phenomena

- Elementary excitations in solids describe the response of a solid to a perturbation
  - Quasiparticles

    usually fermions, resemble the particles that make the system, e.g. quasi-electrons
  - Collective excitations

    usually bosons, describe collective motions

    use second quantization with Fermi-Dirac or Bose-Einstein statistics

Magnetism

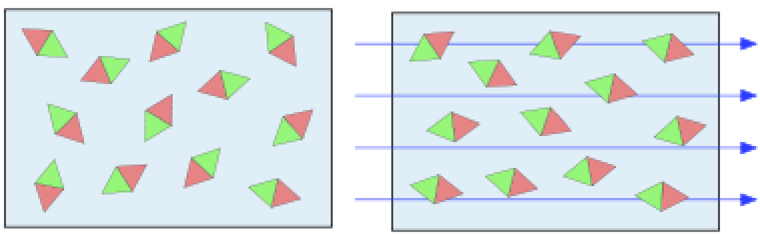
- the Bohr–van Leeuwen theorem

  when statistical mechanics and classical mechanics are applied consistently, the thermal average of the magnetization is always zero.
- Magnetism in solids is solely a quantum mechanical effect
- Origin of the magnetic moment:
  - Electron spin $\vec{S}$
  - Electron orbital momentum $\vec{L}$
- From (macroscopic) response to external magnetic field $\vec{H}$
  - Diamagnetism $\chi < 0$, $\chi \sim 1 \times 10^{-6}$, insensitive to temperature
  - Paramagnetism $\chi > 0$, $\chi = \frac{C}{T}$ Curie law

    $\chi = \frac{C}{T+\Delta}$ Curie-Weiss law
  - Ferromagnetism exchange interaction (quantum)

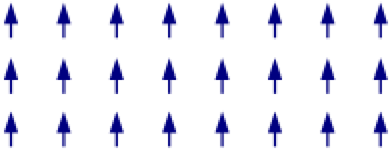
物質的磁性分類

巨觀： 順磁性
Paramagnetism

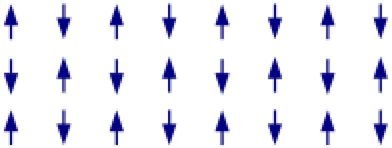


逆磁性
diamagnetism

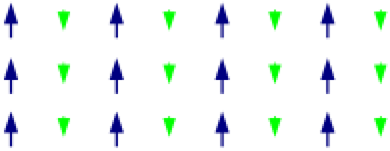
微觀： 鐵磁性
Ferromagnetism

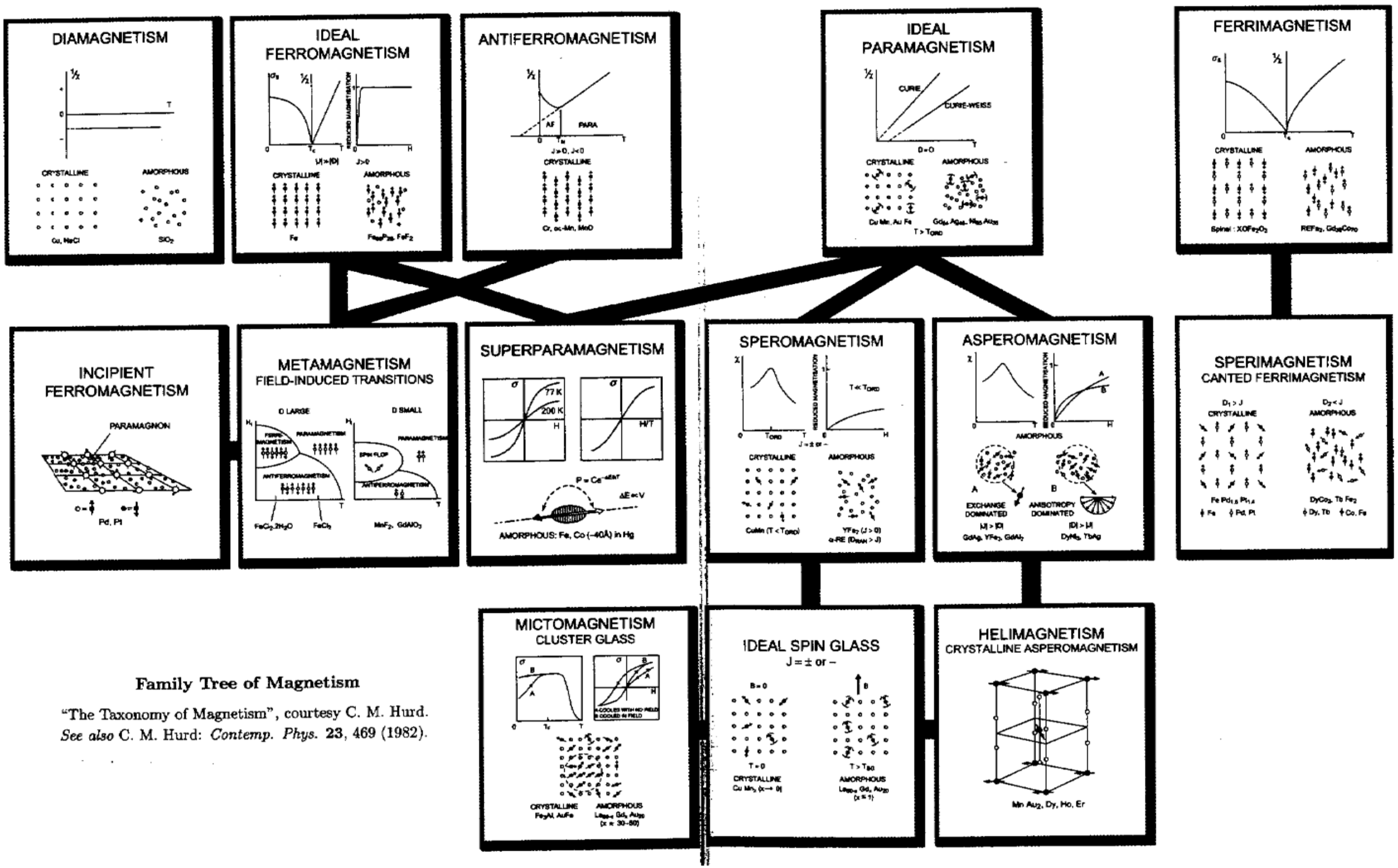


反鐵磁性
Antiferromagnetism



亞鐵磁性
Ferrimagnetism

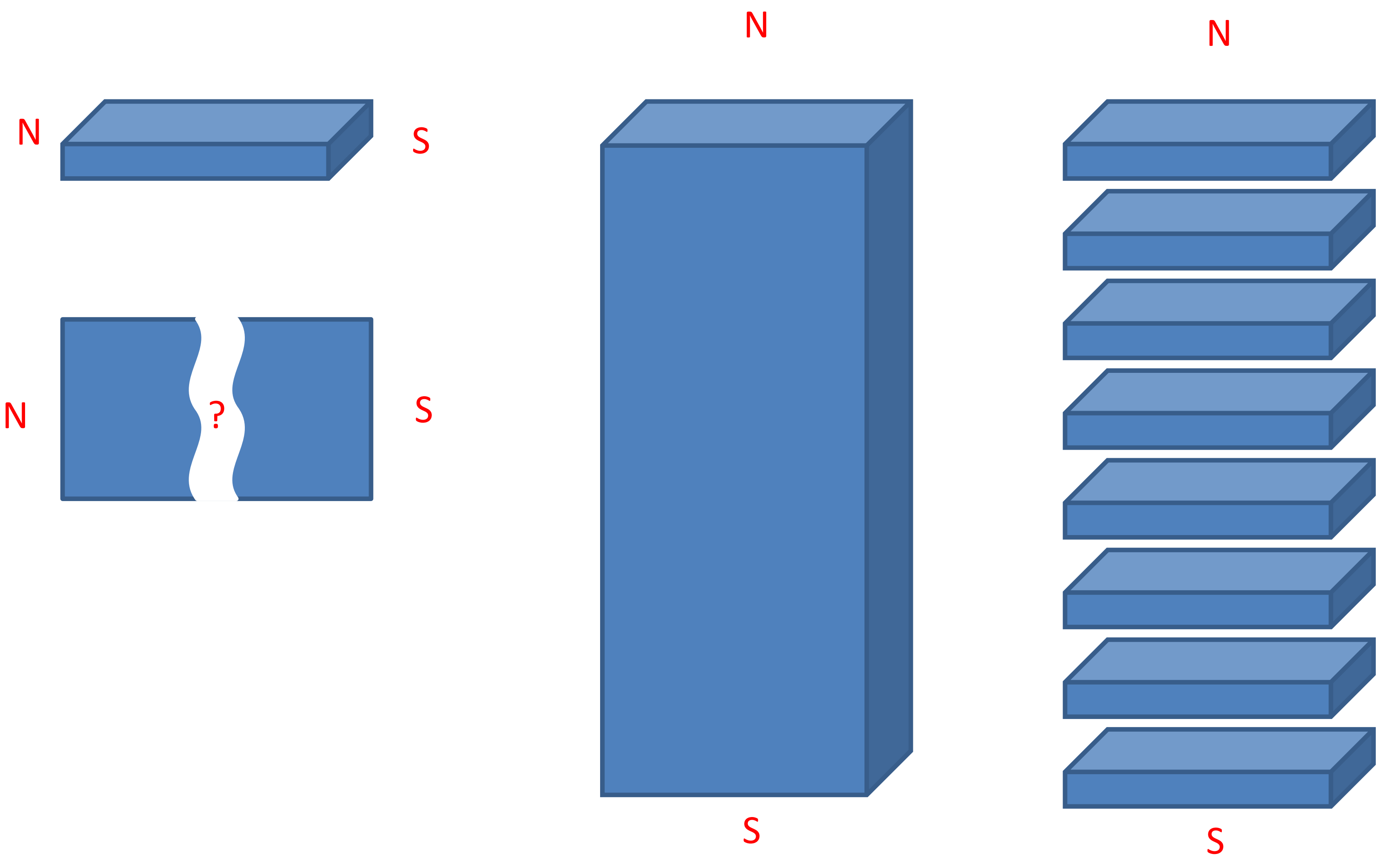




Family Tree of Magnetism

"The Taxonomy of Magnetism", courtesy C. M. Hurd.
See also C. M. Hurd: *Contemp. Phys.* **23**, 469 (1982).

為什麼(大部分)磁鐵打破後會相斥?



- Classical and quantum theory for diamagnetism
  - Calculate $\langle r^2 \rangle$
- Classical and quantum theory for paramagnetism
  - Superparamagnetism, Langevin function
  - Hund's rules
  - Magnetic state ${}^{2S+1}L_J$
  - Crystal field
  - Quenching of orbital angular momentum L_z
    - **Angular momentum operator**
    - **Spherical harmonics**
  - Jahn-Teller effect
  - Paramagnetic susceptibility of conduction electrons

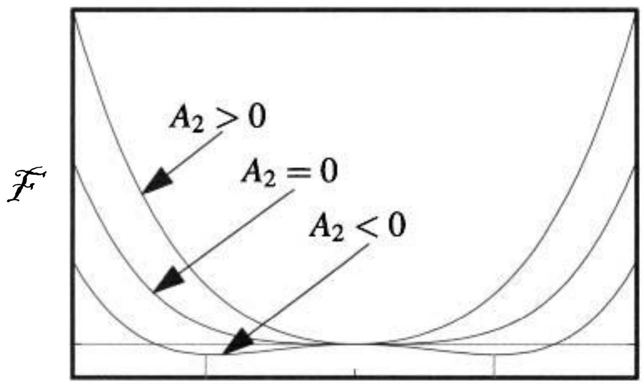
- Ferromagnetism
  - Microscopic – ferro, antiferro, ferri magnetism
  - Exchange interaction
  - Exchange splitting – source of magnetization

    two-electron system spin-independent Schrodinger equation
  - Type of exchange: direct exchange, super exchange, indirect exchange, itinerant exchange
  - Spin Hamiltonian and Heisenberg model
  - Molecular-field (mean-field) approximation

Critical phenomena

Universality. Divergences near the critical point are identical in a variety of apparently different physical systems and also in a collection of simple models.
Scaling. The key to understanding the critical point lies in understanding the relationship between systems of different sizes. Formal development of this idea led to the *renormalization group* of Wilson (1975).

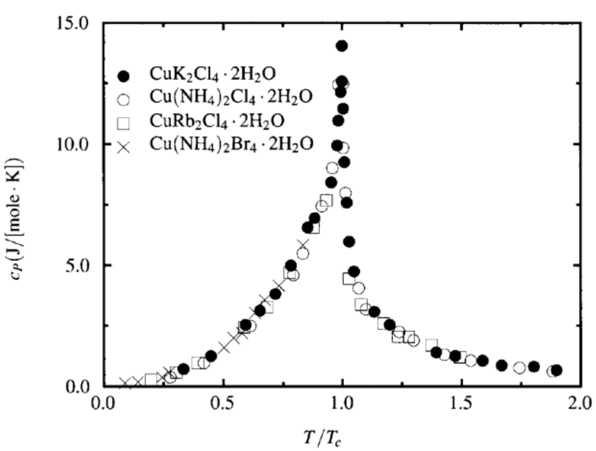
Landau Free Energy



$$\mathcal{F}(M, T) = A_0(T) + A_2(T)M^2 + A_4(T)M^4 + HM.$$

$$t \equiv \frac{T - T_C}{T_C}$$

$$\mathcal{F} = a_2 t M^2 + a_4 M^4 + HM.$$



Molar heat capacities of four ferromagnetic copper salts versus scaled temperature T/T_c. [Source Jongh and Miedema (1974).]

Correspondence between Liquids and Magnets

- ***Specific Heat***— α
- ***Magnetization and Density***— β
- ***Compressibility and** Susceptibility*— γ
- ***Critical Isotherm***— δ
- ***Correlation Length*** — ν
- ***Power-Law Decay at Critical Point***— η

Summary of critical exponents, showing correspondence between fluid-gas systems, magnetic systems, and the three-dimensional Ising model.

Exponent	Fluid	Magnet	Mean Field Theory	Experiment	3d Ising
α	$C_V \sim \|t\|^{-\alpha}$	$C_V \sim \|t\|^{-\alpha}$	discontinuity	0.11–0.12	0.110
β	$\Delta n \sim \|t\|^{\beta}$	$M \sim \|t\|^{\beta}$	$\frac{1}{2}$	0.35–0.37	0.325
γ	$K_T \sim \|t\|^{-\gamma}$	$\chi \sim \|t\|^{-\gamma}$	1	1.21–1.35	1.241
δ	$P \sim \|\Delta n\|^{\delta}$	$\|H\| \sim \|M\|^{\delta}$	3	4.0–4.6	4.82
ν	$\xi \sim \|t\|^{-\nu}$	$\xi \sim \|t\|^{-\nu}$		0.61–0.64	0.63
η	$g(r) \sim r^{-1-\eta}$	$g(r) \sim r^{-1-\eta}$		0.02–0.06	0.032

Source: Vicentini-Missoni (1972) p. 67, Cummins (1971), p. 417, and Goldenfeld (1992) p. 384.

Relations Among Exponents

$\alpha + 2\beta + \gamma = 2$ $\qquad$ $(2 - \eta)\nu = \gamma$

$\delta = 1 + \frac{\gamma}{\beta}$ $\qquad$ $2 - \alpha = 3\nu$

- Stoner band ferromagnetism

  Teodorescu, C. M.; Lungu, G. A. (November 2008). "Band ferromagnetism in systems of variable dimensionality". *Journal of Optoelectronics and Advanced Materials* **10** (11): 3058–3068.

$$\mathcal{E} = \int_0^{\mathcal{E}_F-\Delta} d\mathcal{E}'\, D(\mathcal{E}')\mathcal{E}' + \frac{1}{2}\int_{\mathcal{E}_F-\Delta}^{\mathcal{E}_F+\Delta} d\mathcal{E}'\, D(\mathcal{E}')\mathcal{E}' - \frac{1}{2}nJ\,\langle S\rangle^2$$

$$\langle S\rangle = \frac{1}{2n}\int_{\mathcal{E}_F-\Delta}^{\mathcal{E}_F+\Delta} d\mathcal{E}'\, \frac{1}{2}D(\mathcal{E}') = \frac{1}{2n}D(\mathcal{E}_F)\Delta$$

$$\left.\frac{\partial\mathcal{E}}{\partial\Delta}\right|_{\mathcal{E}_F} = \Delta D(\mathcal{E}_F) - \frac{J}{4n}D(\mathcal{E}_F)^2\Delta$$

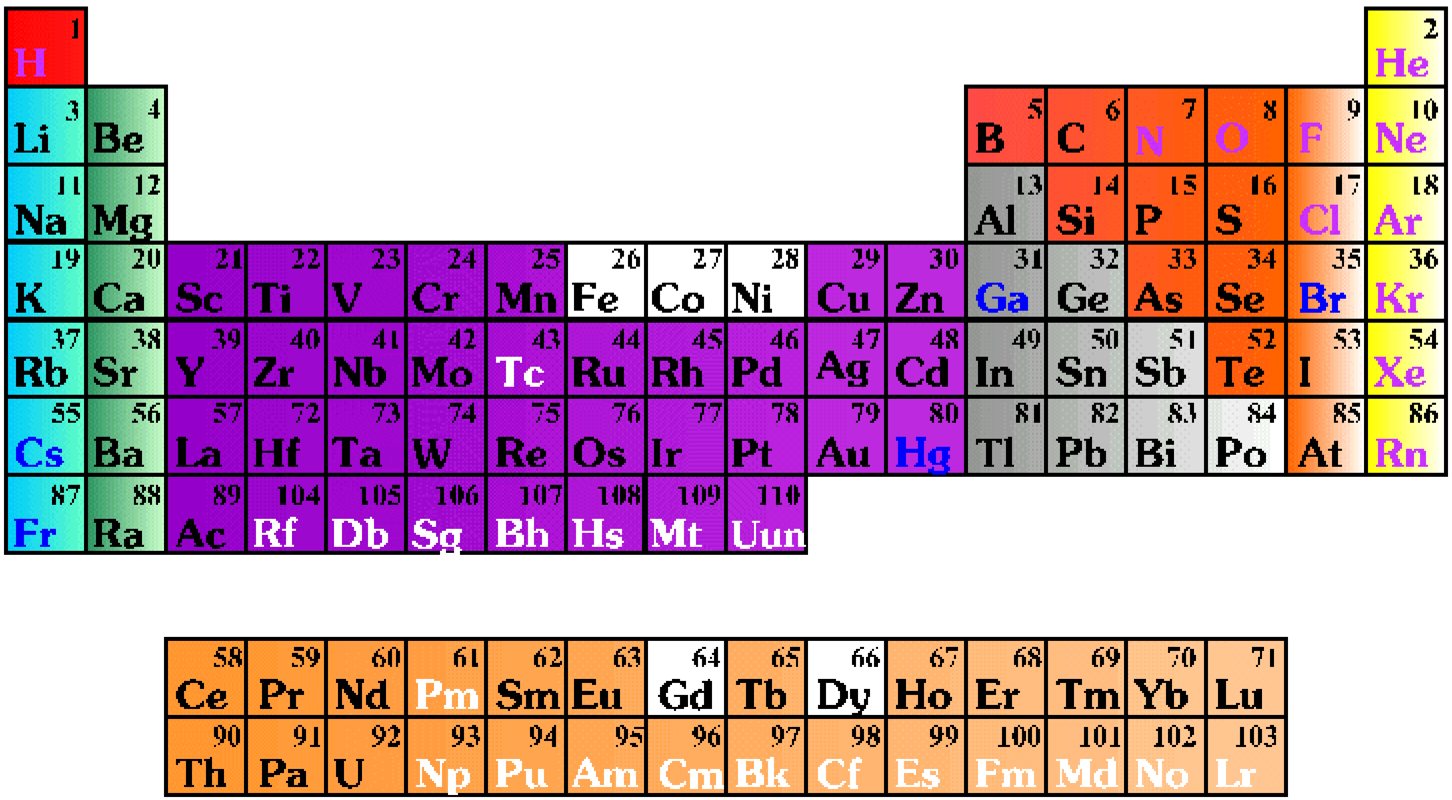
$$\left.\frac{\partial\mathcal{E}}{\partial\Delta}\right|_{\mathcal{E}_F} = 0 \Rightarrow \frac{J}{n}D(\mathcal{E}_F) = 4$$

鐵磁性元素 : 鐵 Fe, 鈷 Co, 鎳 Ni, 釓 Gd, 鏑 Dy,
錳 Mn, 鈀 Pd ??

Elements with ferromagnetic properties

合金, alloys

錳氧化物 MnOx



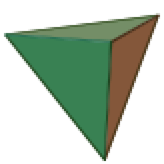
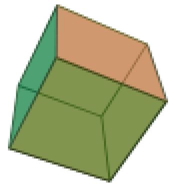
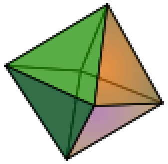
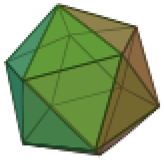
Platonic solid

From Wikipedia

In geometry, a Platonic solid is a convex polyhedron that is regular, in the sense of a regular polygon. Specifically, the faces of a Platonic solid are congruent regular polygons, with the same number of faces meeting at each vertex; thus, all its edges are congruent, as are its vertices and angles.
There are precisely five Platonic solids (shown below):

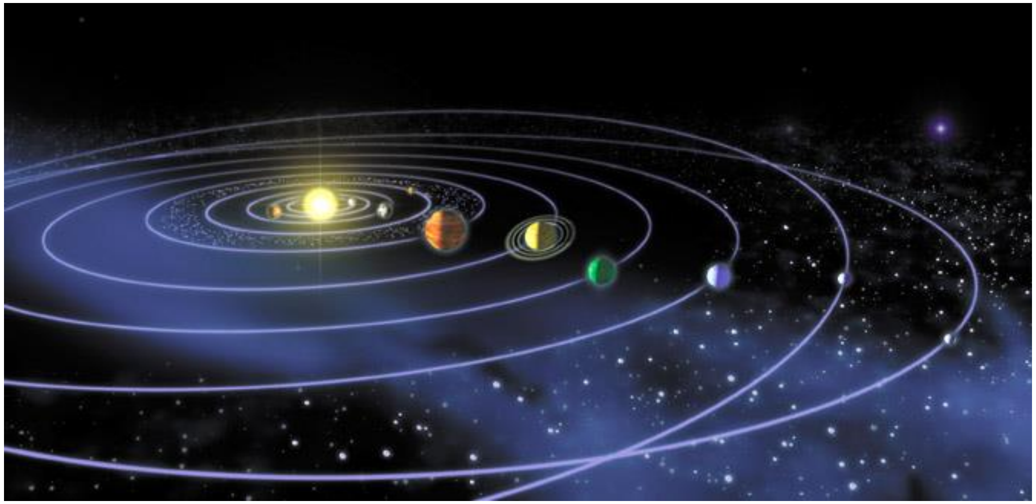
The name of each figure is derived from its number of faces: respectively 4, 6, 8, 12, and 20.

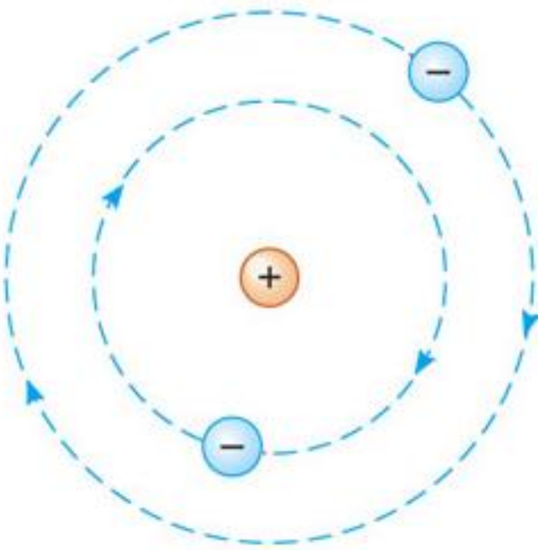
The aesthetic beauty and symmetry of the Platonic solids have made them a favorite subject of geometers for thousands of years. They are named for the ancient Greek philosopher Plato who theorized that the classical elements were constructed from the regular solids.

				
Tetrahedron	Cube hexahedron	Octahedron	Dodecahedron	Icosahedron

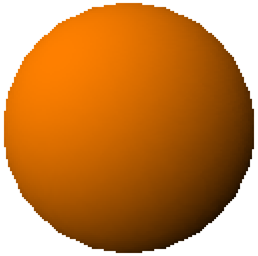
Solar system

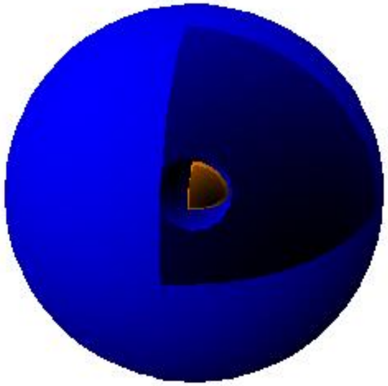
Electronic orbit

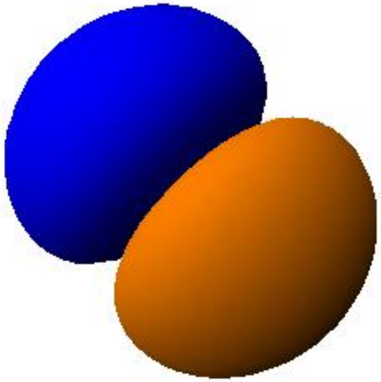




s, p electron orbits



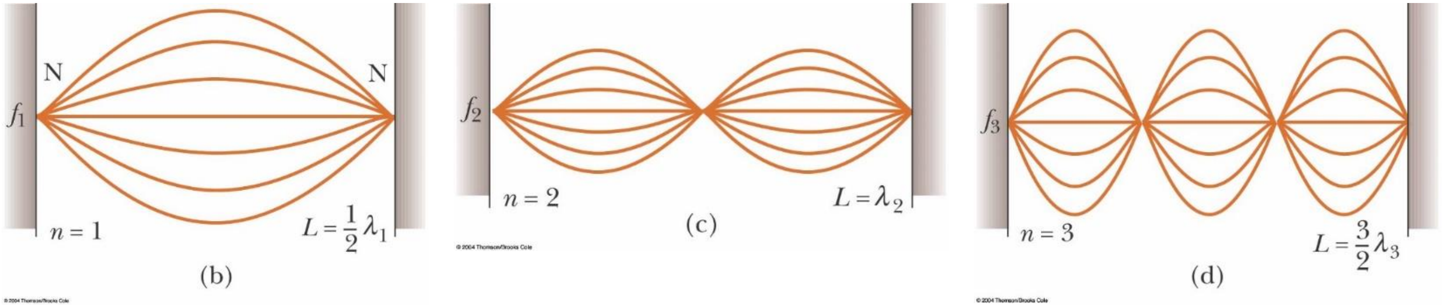




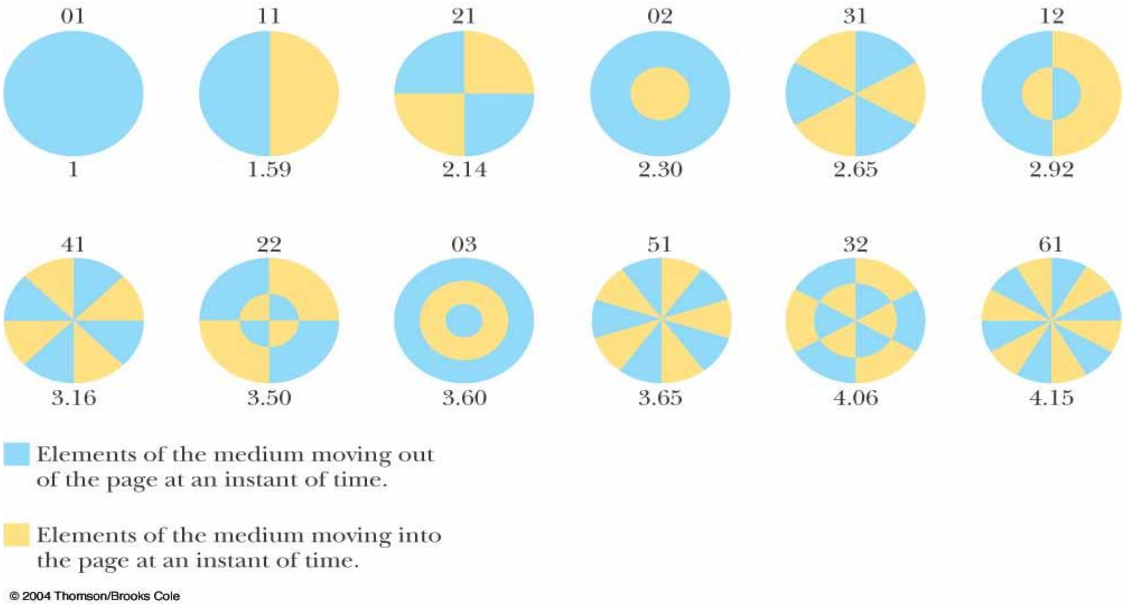
Orbital viewer

Resonance

One-dimensional



Two-dimensional



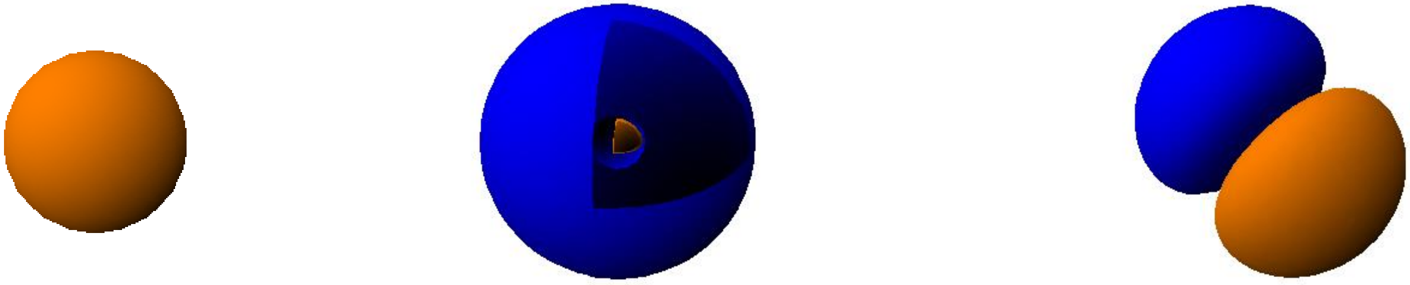
Three-dimensional

Hydrogen atom

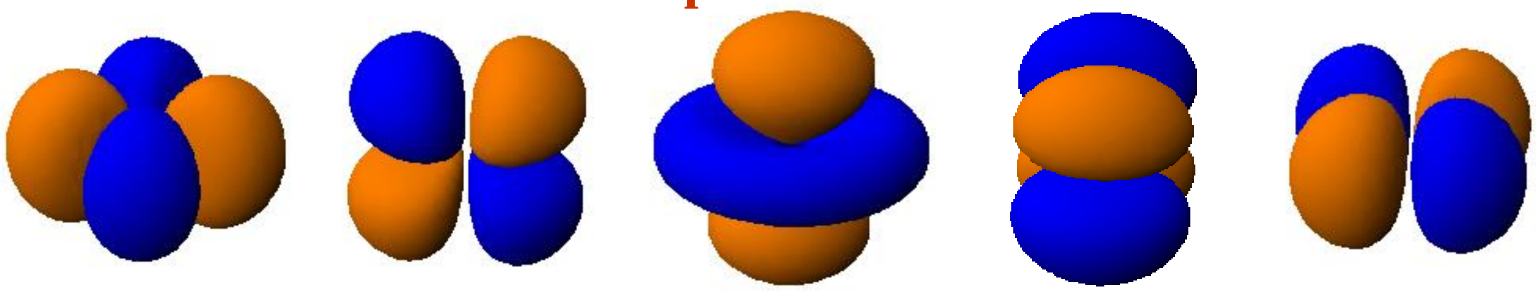
3d transition metals:

Mn atom has 5 d ↑ electrons   Bulk Mn is NOT magnetic

s, p electron orbital



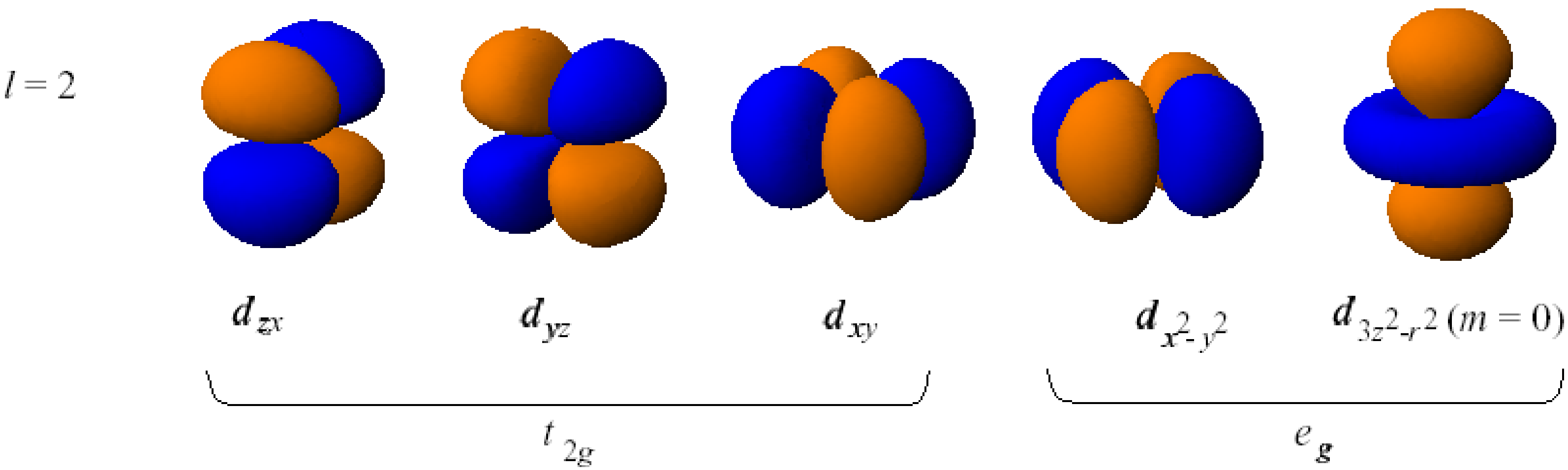
3d electron distribution in real space

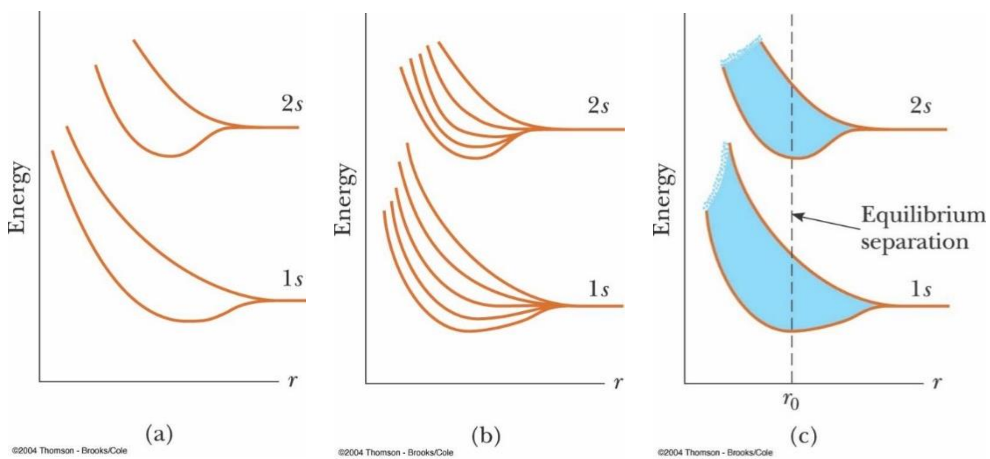


Co atom has 5 d ↑electrons and 2 d ↓ electrons
Bulk Co is magnetic.

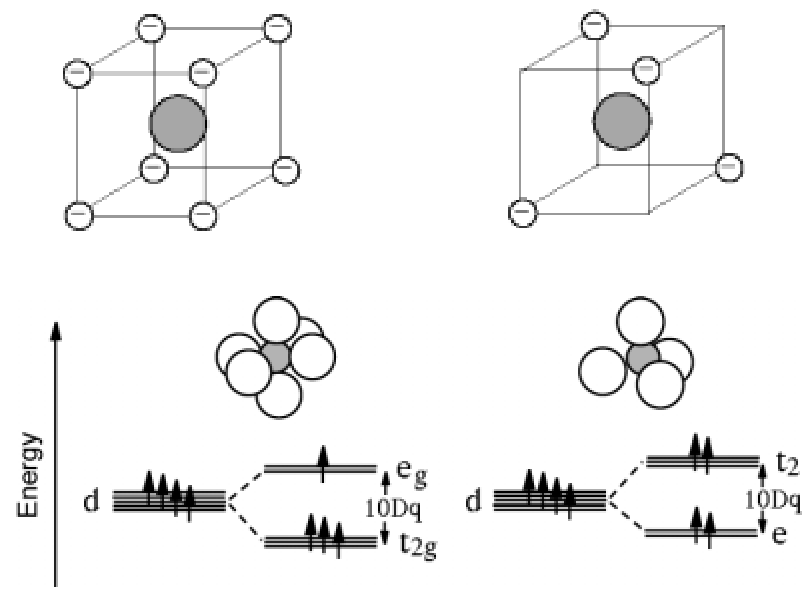
Orbital viewer

d orbitals

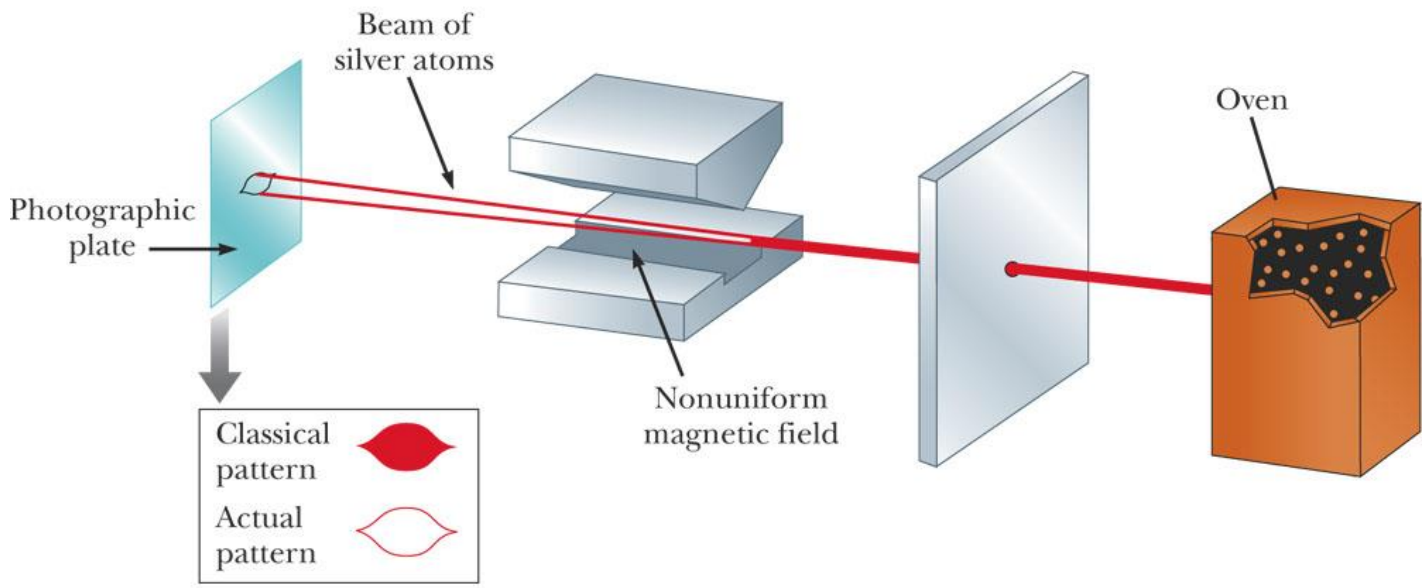




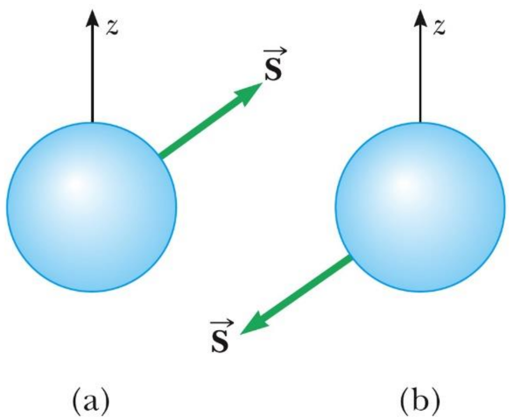
Crystal-field splitting



Stern-Gerlach Experiment



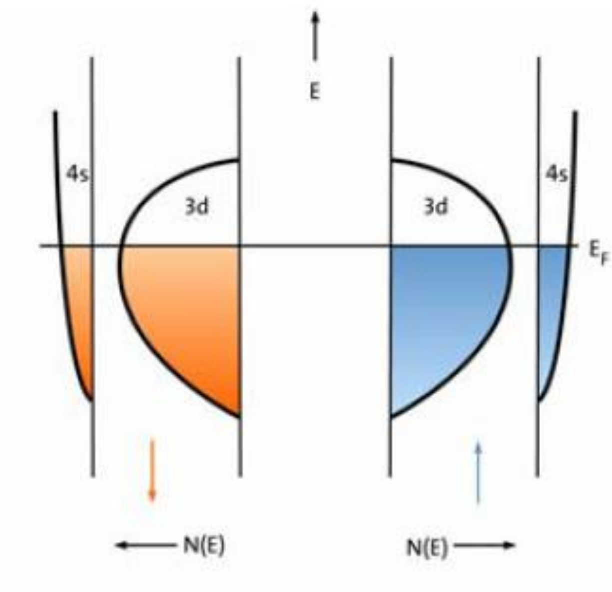
© 2006 Brooks/Cole - Thomson



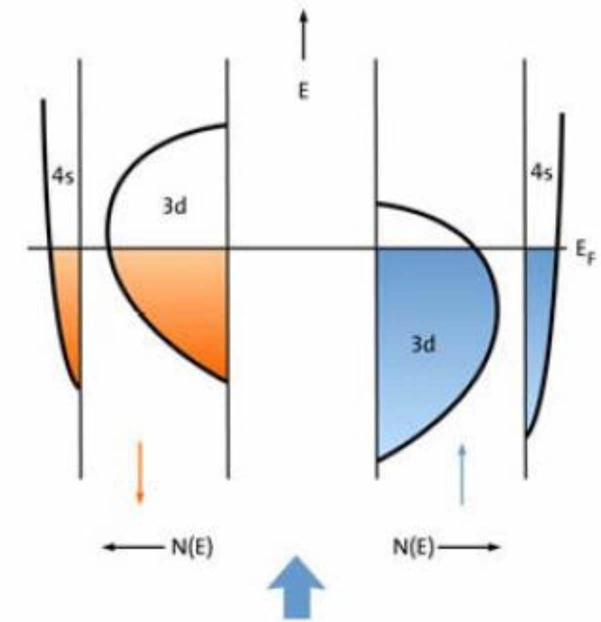
There are two kinds of electrons:
spin-up and spin-down.

Stoner criterion for ferromagnetism:

I $N(E_F) > 1$, I is the **Stoner exchange parameter** and $N(E_F)$ is the density of states at the Fermi energy.



For the non-magnetic state there are identical density of states for the two spins.



For a ferromagnetic state, $N\uparrow > N\downarrow$. The polarization is indicated by the thick blue arrow.

Schematic plot for the energy band structure of 3d transition metals.

Teodorescu and Lungu, "Band ferromagnetism in systems of variable dimensionality". *J Optoelectronics and Adv. Mat.* **10,** 3058–3068 (2008).

Exchange interaction

Русский
Українська
Tiếng Việt
Edit links

Exchange Interactions between localized electron magnetic moments [edit]

Quantum mechanical particles are classified as bosons or fermions. The spin–statistics theorem of quantum field theory demands that all particles with half-integer spin behave as fermions and all particles with integer spin behave as bosons. Multiple bosons may occupy the same quantum state; by the Pauli exclusion principle, however, no two fermions can occupy the same state. Since electrons have spin 1/2, they are fermions. This means that the overall wave function of a system must be antisymmetric when two electrons are exchanged, i.e. interchanged with respect to both spatial and spin coordinates. First, however, exchange will be explained with the neglect of spin.

Exchange of spatial coordinates [edit]

Taking a hydrogen molecule-like system (i.e. one with two electrons), we may attempt to model the state of each electron by first assuming the electrons behave independently, and taking wave functions in position space of $\Phi_a(r_1)$ for the first electron and $\Phi_b(r_2)$ for the second electron. We assume that Φ_a and Φ_b are orthogonal, and that each corresponds to an energy eigenstate of its electron. Now, we may construct a wave function for the overall system in position space by using an antisymmetric combination of the product wave functions in position space:

$$\Psi_A(\vec{r}_1,\vec{r}_2) = \frac{1}{\sqrt{2}}[\Phi_a(\vec{r}_1)\Phi_b(\vec{r}_2) - \Phi_b(\vec{r}_1)\Phi_a(\vec{r}_2)] \quad (1)$$

Alternatively, we may also construct the overall position–space wave function by using a symmetric combination of the product wave functions in position space:

$$\Psi_S(\vec{r}_1,\vec{r}_2) = \frac{1}{\sqrt{2}}[\Phi_a(\vec{r}_1)\Phi_b(\vec{r}_2) + \Phi_b(\vec{r}_1)\Phi_a(\vec{r}_2)] \quad (2)$$

Treating the exchange interaction in the hydrogen molecule by the perturbation method, the overall Hamiltonian is:

$\mathcal{H} = \mathcal{H}^{(0)} + \mathcal{H}^{(1)}$

where $\mathcal{H}^{(0)} = -\frac{\hbar^2}{2m}\left(\nabla_1^2 + \nabla_2^2\right) - \frac{e^2}{r_1} - \frac{e^2}{r_2}$ and $\mathcal{H}^{(1)} = \left(\frac{e^2}{R_{ab}} + \frac{e^2}{r_{12}} - \frac{e^2}{r_{a1}} - \frac{e^2}{r_{b2}}\right)$

Two eigenvalues for the system energy are found:

$$E_{+/-} = E_{(0)} + \frac{C \pm J_{ex}}{1 \pm B^2} \quad (3)$$

where the E_+ is the spatially symmetric solution and E_- is the spatially antisymmetric solution. A variational calculation yields similar results. $\mathcal{H}$ can be diagonalized by using the position–space functions given by Eqs. (1) and (2). In Eq. (3), C is the **Coulomb integral**, B is the **overlap integral**, and J_{ex} is the **exchange integral**. These integrals are given by:

$$C = \int \Phi_a(\vec{r}_1)^2 \left(\frac{1}{R_{ab}} + \frac{1}{r_{12}} - \frac{1}{r_{a1}} - \frac{1}{r_{b2}}\right) \Phi_b(\vec{r}_2)^2 \, d^3r_1 \, d^3r_2 \quad (4)$$

$$B = \int \Phi_b(\vec{r}_2)\Phi_a(\vec{r}_2) \, d^3r_2 \quad (5)$$

$$J_{ex} = \int \Phi_a(\vec{r}_1)\Phi_b(\vec{r}_2) \left(\frac{1}{R_{ab}} + \frac{1}{r_{12}} - \frac{1}{r_{a1}} - \frac{1}{r_{b2}}\right) \Phi_b(\vec{r}_1)\Phi_a(\vec{r}_2) \, d^3r_1 \, d^3r_2 \quad (6)$$

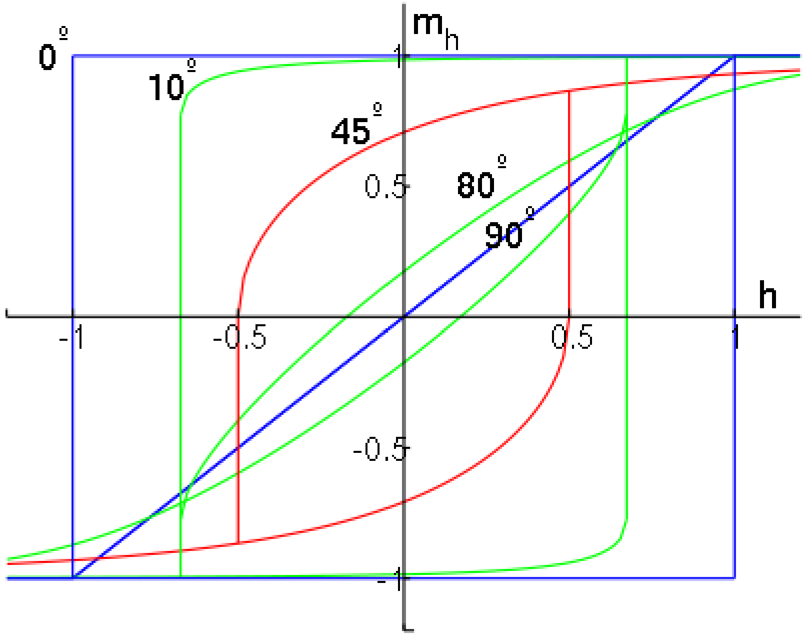
The terms in parentheses in Eqs. (4) and (6) correspond to: proton–proton repulsion (R_{ab}), electron–electron repulsion (r_{12}), and electron–proton attraction ($r_{a1/a2/b1/b2}$).All quantities are assumed to be real.

Although in the hydrogen molecule the exchange integral, Eq. (6), is negative, Heisenberg first suggested that it changes sign at some critical ratio of internuclear distance to

Stoner–Wohlfarth model

The **Stoner–Wohlfarth model** is a widely used model for the magnetization of single-domain ferromagnets.[1] It is a simple example of magnetic hysteresis and is useful for modeling small magnetic particles in magnetic storage, biomagnetism, rock magnetism and paleomagnetism.

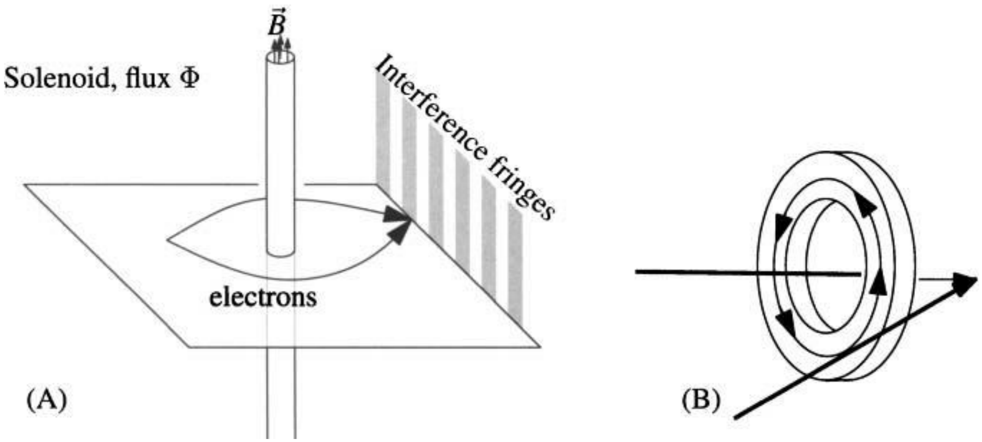
$$E = K_u V \sin^2(\phi - \theta) - \mu_0 M_s V H \cos\phi,$$

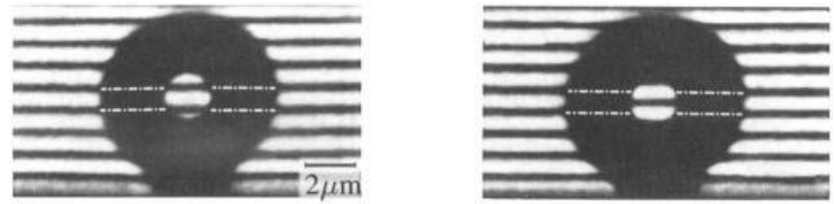


K_u is the anisotropy parameter V is the volume of the magnet, M_s is the saturation magnetization, and μ_0 is the vacuum permeability

Berry Phase

Aharonov-Bohm Effect



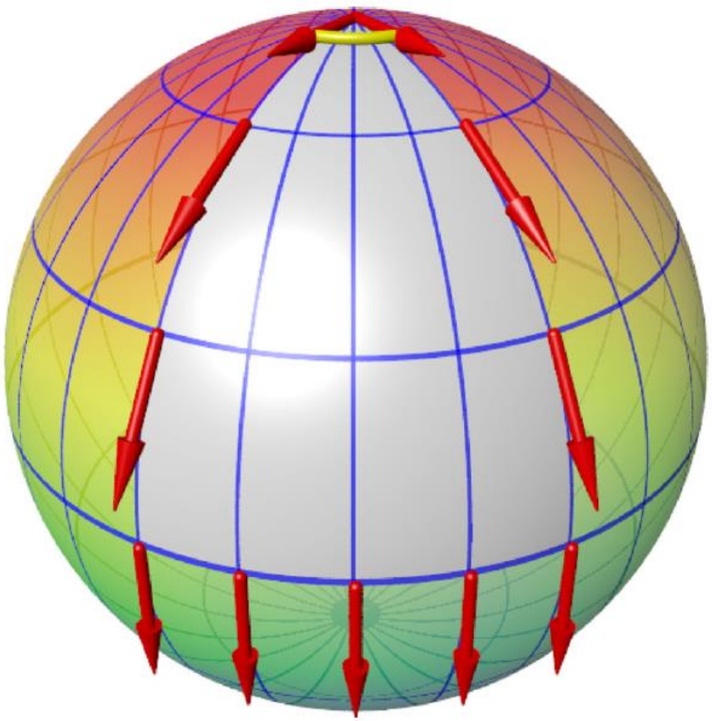


Electrons traveling around a flux tube suffer a phase change and can interfere with themselves even if they only travel through regions where B = 0.
(B) An open flux tube is not experimentally realizable, but a small toroidal magnet with no flux leakage can be constructed instead.

Electron hologram showing interference fringes of electrons passing through small toroidal magnet. The magnetic flux passing through the torus is quantized so as to produce an integer multiple of π phase change in the electron wave functions. The electron is completely screened from the magnetic induction in the magnet. In (A) the phase change is 0, while in (B) the phase change is π. [Source: Tonomura (1993), p. 67.]

$$\Phi = \int d^2r\, B_z = \oint d\vec{r} \cdot \vec{A}$$

$$A_\phi = \frac{\Phi}{2\pi r}$$



Parallel transport of a vector along a closed path on the sphere S_2 leads to a geometric phase between initial and final state.

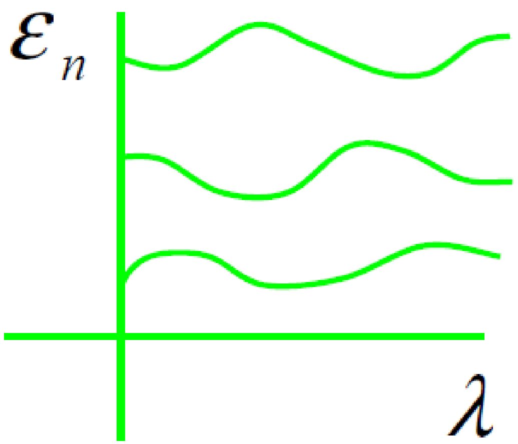
Real-space Berry phases: Skyrmion soccer (invited)
Karin Everschor-Sitte and Matthias Sitte
Journal of Applied Physics **115**, 172602 (2014); doi: 10.1063/1.4870695

Berry phase formalism for intrinsic Hall effects

From Prof. Guo Guang-Yu

Berry phase

[Berry, Proc. Roy. Soc. London A 392, 451 (1984)]

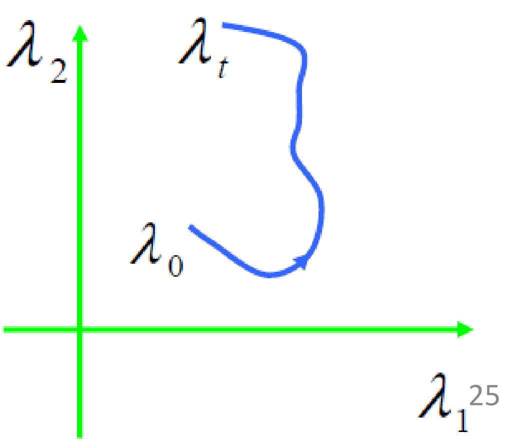


Parameter dependent system:

$$\{\, \varepsilon_n(\lambda),\ \psi_n(\lambda) \,\}$$

Adiabatic theorem:

$$\Psi(t) = \psi_n(\lambda(t))\ e^{-i\int_0^t dt\, \varepsilon_n/\hbar}\ e^{-i\gamma_n(t)}$$



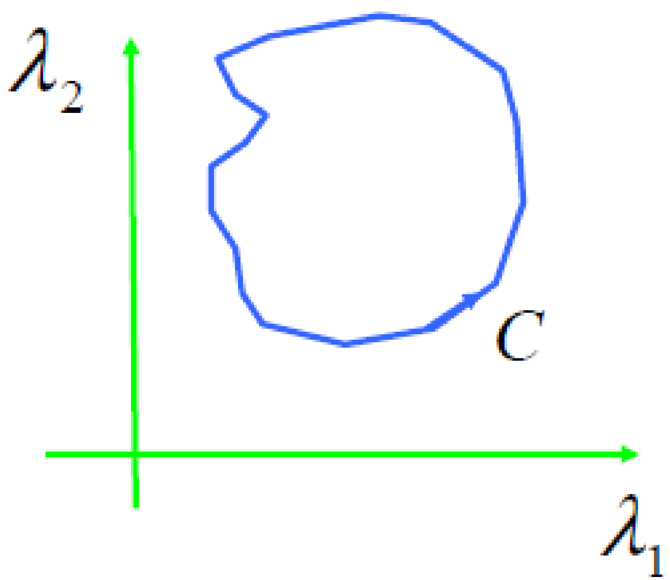
Geometric phase:

$$\gamma_n = \int_{\lambda_0}^{\lambda_t} d\lambda \,\langle \psi_n | \, i\frac{\partial}{\partial \lambda} | \psi_n \rangle$$

Well defined for a closed path

From Prof. Guo Guang-Yu

$$\gamma_n = \oint_C d\lambda \langle \psi_n | i \frac{\partial}{\partial \lambda} | \psi_n \rangle$$



Stokes theorem

$$\gamma_n = \iint d\lambda_1 d\lambda_2 \, \Omega$$

Berry Curvature

$$\Omega = i \frac{\partial}{\partial \lambda_1} \langle \psi | \frac{\partial}{\partial \lambda_2} | \psi \rangle - i \frac{\partial}{\partial \lambda_2} \langle \psi | \frac{\partial}{\partial \lambda_1} | \psi \rangle$$

Analogies

From Prof. Guo Guang-Yu

Berry curvature	Magnetic field
$\Omega(\vec{\lambda})$	$B(\vec{r})$
Berry connection	Vector potential
$\langle \psi \| i \frac{\partial}{\partial \lambda} \| \psi \rangle$	$A(\vec{r})$
Geometric phase	Aharonov-Bohm phase
$\oint d\lambda \langle \psi \| i \frac{\partial}{\partial \lambda} \| \psi \rangle = \iint d^2\lambda \ \Omega(\vec{\lambda})$	$\oint dr \ A(\vec{r}) = \iint d^2 r \ B(\vec{r})$
Chern number	Dirac monopole
$\oiint d^2\lambda \ \Omega(\vec{\lambda}) = \text{integer}$	$\oiint d^2 r \ B(\vec{r}) = \text{integer } h/e$

Semiclassical dynamics of Bloch electrons

Old version [e.g., Aschroft, Mermin, 1976]

From Prof. Guo Guang-Yu

$$\dot{\mathbf{x}}_c = \frac{1}{\hbar}\frac{\partial \varepsilon_n(\mathbf{k})}{\partial \mathbf{k}},$$

$$\dot{\mathbf{k}} = -\frac{e}{\hbar}\mathbf{E} - \frac{e}{\hbar}\dot{\mathbf{x}}_c \times \mathbf{B} = \frac{e}{\hbar}\frac{\partial \varphi(\mathbf{r})}{\partial \mathbf{r}} - \frac{e}{\hbar}\dot{\mathbf{x}}_c \times \mathbf{B}$$

New version [Marder, 2000]

Berry phase correction [Chang & Niu, PRL (1995), PRB (1996)]

$$\dot{\mathbf{x}}_c = \frac{1}{\hbar}\frac{\partial \varepsilon_n(\mathbf{k})}{\partial \mathbf{k}} - \dot{\mathbf{k}} \times \mathbf{\Omega}_n(\mathbf{k}),$$

$$\dot{\mathbf{k}} = \frac{e}{\hbar}\frac{\partial \varphi(\mathbf{r})}{\partial \mathbf{r}} - \frac{e}{\hbar}\dot{\mathbf{x}}_c \times \mathbf{B},$$

$$\mathbf{\Omega}_n(\mathbf{k}) = -\,\mathrm{Im}\left\langle \frac{\partial u_{n\mathbf{k}}}{\partial \mathbf{k}} \middle| \times \middle| \frac{\partial u_{n\mathbf{k}}}{\partial \mathbf{k}} \right\rangle \qquad \text{(Berry curvature)}$$

Demagnetization factor D

can be solved analytically in some cases, numerically in others

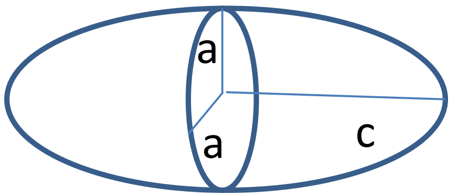
For an ellipsoid $D_x + D_y + D_z = 1$ (SI units) $D_x + D_y + D_z = 4\pi$ (cgs units)

Solution for Spheroid $a = b \neq c$

1. Prolate spheroid (football shape) $c/a = r > 1$; $a = b$, In cgs units

$$D_c = \frac{4\pi}{r^2-1}\left[\frac{r}{\sqrt{r^2-1}}\ln\left(r+\sqrt{r^2-1}\right)-1\right]$$

$$D_a = D_b = \frac{4\pi - D_c}{2}$$

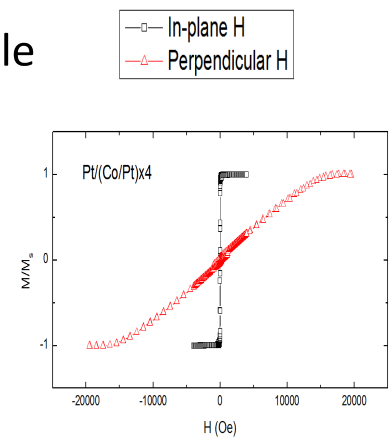


Limiting case r >> 1 (long rod)

$$D_c = \frac{4\pi}{r^2}\left[\ln(2r)-1\right] \ll 1$$

$$D_a = D_b = 2\pi$$

Note: you measure $2\pi M$ *without* knowing the sample

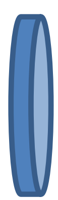


2. Oblate Spheroid (pancake shape) $c/a = r < 1$; $a = b$

$$D_c = \frac{4\pi}{1-r^2}\left[1-\frac{r}{\sqrt{1-r^2}}\cos^{-1} r\right]$$

$$D_a = D_b = \frac{4\pi - D_c}{2}$$

Limiting case r >> 1 (flat disk)



$$D_c = 4\pi$$

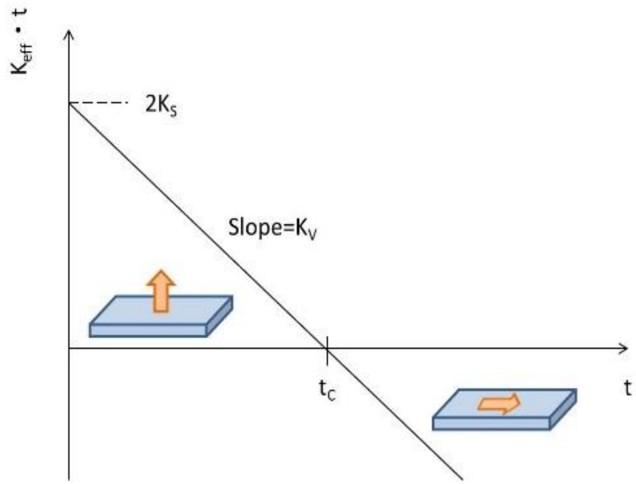
$$D_a = D_b = \pi^{2} r \ll 1$$

Note: you measure $4\pi M$ *without* knowing the sample

Surface anisotropy

$$E = E_{exchange} + E_{Zeeman} + E_{mag} + E_{anisotropy} + \cdots$$

- $E_{ex} : \sum 2J\overrightarrow{S_i} \cdot \overrightarrow{S_j}$
- $E_{Zeeman} : \vec{M} \cdot \vec{H}$
- $E_{mag} : \frac{1}{8\pi}\int B^2 dV$
- $E_{anisotropy}$



For hcp Co$= K_1' \sin^2\theta + K_2{}' \sin^4\theta$

For bcc Fe $= K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2) + K_2(\alpha_1^2\alpha_2^2\alpha_3^2)$

α_i : directional cosines

Surface anisotropy $K_{\text{eff}} = \frac{2K_S}{t} + K_V$ ➔ $K_{\text{eff}} \cdot t = 2K_S + K_V \cdot t$

Ferromagnetic domains

- competition between exchange, anisotropy, and magnetic energies.
- Bloch wall: rotation out of the plane of the two spins
- Neel wall: rotation within the plane of the two spins

For a 180° Bloch wall rotated in N+1 atomic planes $N\Delta E_{ex} = N(JS^2\left(\frac{\pi}{N}\right)^2)$

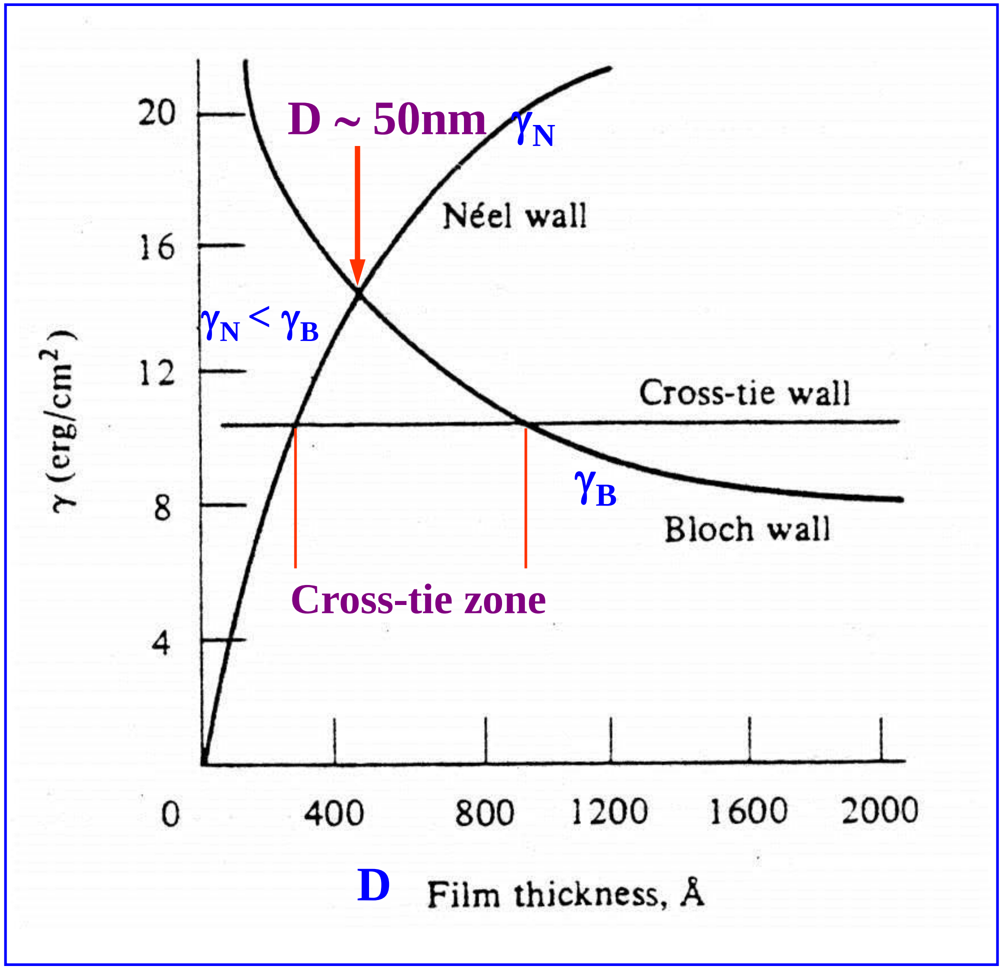
Wall energy density $\sigma_w = \sigma_{ex} + \sigma_{anis} \approx JS^2\pi^2/(Na^2) + KNa$ a : lattice constant

$\partial\sigma_w/\partial N \equiv 0,$ $N = \sqrt{[JS^2\pi^2/(Ka^3)]} \approx 300$ in Fe

$\sigma_w = 2\pi\sqrt{KJS^2/a} \approx$ 1 erg/cm^2 in Fe

Wall width $Na = \pi\sqrt{JS^2/Ka} \equiv \pi\sqrt{\frac{A}{K}}$, $A = JS^2/a$ Exchange stiffness constant

Domain wall energy γ versus thickness D of $Ni_{80}Fe_{20}$ thin films



$\gamma_N < \gamma_B \sim$ 50nm

Thick films have Bloch walls
Thin films have Neel walls

Cross-tie walls show up in between.

$A = 10^{-6}$ erg/cm

$K = 1500$ erg/cm^3

Magnetic Resonance

- Nuclear Magnetic Resonance (NMR)
  - Line width
  - Hyperfine Splitting, Knight Shift
  - Nuclear Quadrupole Resonance (NQR)
- Ferromagnetic Resonance (FMR)
  - Shape Effect
  - Spin Wave resonance (SWR)
- Antiferromagnetic Resonance (AFMR)
- Electron Paramagnetic Resonance (EPR or ESR)
  - Exchange narrowing
  - Zero-field Splitting
- Maser

What we can learn:

- From absorption fine structure ➔ electronic structure of single defects
- From changes in linewidth ➔ relative motion of the spin to the surroundings
- From resonance frequency ➔ internal magnetic field
- Collective spin excitations

FMR

Equation of motion of a magnetic moment $\boldsymbol{\mu}$ in an external field B_0

$$\frac{\hbar d\boldsymbol{I}}{dt} = \boldsymbol{\mu} \times \boldsymbol{B} \qquad \boldsymbol{\mu} = \gamma\hbar\boldsymbol{I} \qquad \frac{d\boldsymbol{\mu}}{dt} = \gamma\boldsymbol{\mu} \times \boldsymbol{B} \qquad \frac{d\boldsymbol{M}}{dt} = \gamma\boldsymbol{M} \times \boldsymbol{B}$$

Landau-Lifshitz-Gilbert (LLG) equation

$$\frac{d\boldsymbol{M}}{dt} = -\gamma\boldsymbol{M} \times \boldsymbol{H}_{\text{eff}} + \alpha\boldsymbol{M} \times \frac{d\boldsymbol{M}}{dt}$$

Shape effect:

internal magnetic field

$$B_x^i = B_x^0 - N_x M_x \qquad B_y^i = B_y^0 - N_y M_y \qquad B_z^i = B_z^0 - N_z M_z$$

$$\frac{dM_x}{dt} = \gamma\left(M_y B_z^i - M_z B_y^i\right) = \gamma[B_0 + \left(N_y - N_z\right)M]M_y$$

$$\frac{dM_y}{dt} = \gamma[M(-N_x M_x) - M_x(B_0 - N_z M)] = -\gamma[B_0 + (N_x - N_z)M]M_x$$

To first order $\frac{dM_z}{dt} = 0$ $\qquad M_z = M$

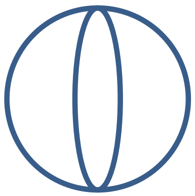
$$\begin{vmatrix} i\omega & \gamma[B_0 + \left(N_y - N_z\right)M] \\ -\gamma[B_0 + (N_x - N_z)M] & i\omega \end{vmatrix} = 0$$

$$\omega_0^2 = \gamma^2[B_0 + \left(N_y - N_z\right)M][B_0 + (N_x - N_z)M]$$

Uniform mode

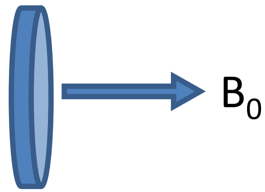
Uniform mode

Sphere



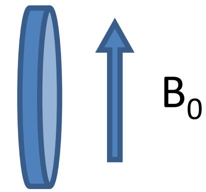
$$N_x = N_y = N_z$$
$$\omega_0 = \gamma\, B_0$$

flat plate with perpendicular field



$$N_x = N_y = 0 \qquad N_z = 4\pi$$
$$\omega_0 = \gamma\,(B_0 - 4\pi M)$$

flat plate with in-plane field



$$N_x = N_z = 0 \qquad N_y = 4\pi$$
$$\omega_0 = \gamma\,[B_0(B_0 + 4\pi M)]^{1/2}$$

Spin wave resonance; Magnons

Consider a one-dimensional spin chain with only nearest-neighbor interactions.

$$U = -2J \sum \vec{S_i} \cdot \vec{S_j}$$ We can derive $$\hbar\omega = 4JS(1 - \cos ka)$$

When $ka << 1$ $\qquad \hbar\omega \cong (2JSa^2)k^2$

flat plate with perpendicular field $\qquad \omega_0 = \gamma\,(B_0 - 4\pi M) + Dk^2$

Quantization of (uniform mode) spin waves, then consider the thermal excitation of Mannons, leads to Bloch $T^{3/2}$ law. $\qquad \Delta M / M(0) \propto T^{3/2}$

AFMR

Spin wave resonance; Antiferromagnetic Magnons

Consider a one-dimensional antiferromangetic spin chain with only nearest-neighbor interactions. Treat sublattice A with up spin S and sublattice B with down spin –S, J<0.

$U = -2J \sum \vec{S_i} \cdot \vec{S_j}$ We can derive $\hbar\omega = -4JS|\sin ka|$

When *ka* << 1 $\hbar\omega \cong (-4JS)|ka|$

AFMR

exchange plus anisotropy fields on the two sublattices

$\boldsymbol{B}_1 = -\lambda \boldsymbol{M}_2 + B_A \hat{\boldsymbol{z}}$ on $\mathbf{M}_1$ $\boldsymbol{B}_2 = -\lambda \boldsymbol{M}_1 - B_A \hat{\boldsymbol{z}}$ on $\mathbf{M}_2$

$M_1^z \equiv M$ $M_2^z \equiv -M$ $M_1^+ \equiv M_1^x + iM_1^y$ $M_2^+ \equiv M_2^x + iM_2^y$ $B_E \equiv \lambda M$

$$\frac{dM_1^+}{dt} = -i\gamma[M_1^+(B_A + B_E) + M_2^+ B_E]$$

$$\frac{dM_2^+}{dt} = -i\gamma[M_2^+(B_A + B_E) + M_1^+ B_E]$$

$$\begin{vmatrix} \gamma(B_A + B_E) - \omega & \gamma B_E \\ B_E & \gamma(B_A + B_E) + \omega \end{vmatrix} = 0$$

$\omega_0^2 = \gamma^2 B_A(B_A + 2B_E)$ Uniform mode

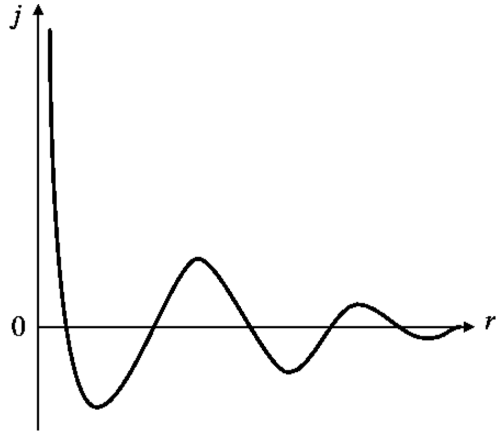
Spintronics

Electronics with electron spin as an extra degree of freedom
Generate, inject, process, and detect spin currents

- Generation: ferromagnetic materials, spin Hall effect, spin pumping effect etc.
- Injection: interfaces, heterogeneous structures, tunnel junctions
- Process: spin transfer torque
- Detection: Giant Magnetoresistance, Tunneling MR
- Historically, from magnetic coupling to transport phenomena

important materials: CoFe, CoFeB, Cu, Ru, IrMn, PtMn, MgO, Al2O3, Pt, Ta

RKKY (*Ruderman-Kittel-Kasuya-Yosida*) interaction



coupling coefficient

$$j\left(\boldsymbol{R}_l - \boldsymbol{R}_{l'}\right) = 9\pi\left(\frac{j^2}{\epsilon_F}\right) F\left(2k_F\left|\boldsymbol{R}_l - \boldsymbol{R}_{l'}\right|\right)$$

$$F(x) = \frac{x\cos x - \sin x}{x^4}$$

Magnetic coupling in superlattices

- Long-range incommensurate magnetic order in a Dy-Y multilayer

  M. B. Salamon, Shantanu Sinha, J. J. Rhyne, J. E. Cunningham, Ross W. Erwin, Julie Borchers, and C. P. Flynn, Phys. Rev. Lett. **56**, 259 - 262 (1986)

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  C. F. Majkrzak, J. W. Cable, J. Kwo, M. Hong, D. B. McWhan, Y. Yafet, and J. V. Waszczak,C. Vettier, Phys. Rev. Lett. **56**, 2700 - 2703 (1986)

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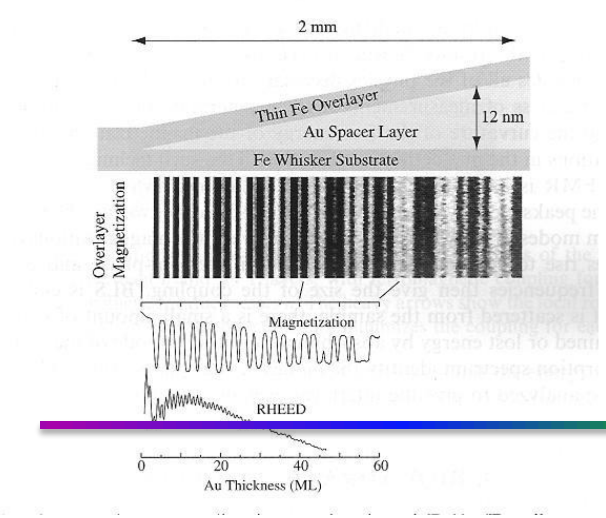


Coupling in wedge-shaped

Fe/Cr/Fe

Fe/Au/Fe

Fe/Ag/Fe

J. Unguris, R. J. Celotta, and D. T. Pierce

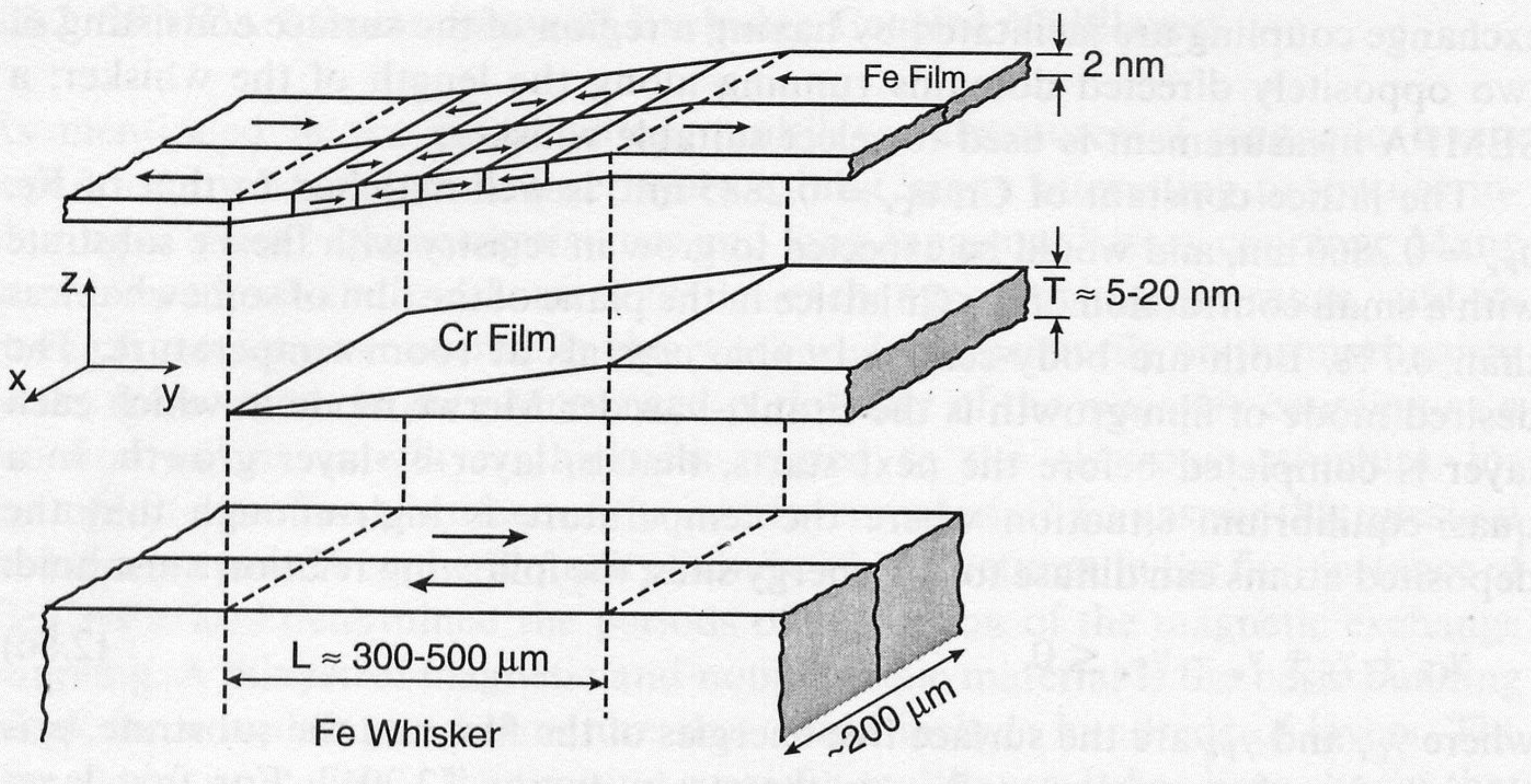


Fig. 2.41. A schematic expanded view of the sample structure showing the Fe(0 0 1) single-crystal whisker substrate, the evaporated Cr wedge, and the Fe overlayer. The arrows in the Fe show the magnetization direction in each domain. The z-scale is expanded approximately 5000 times. (From [2.206])

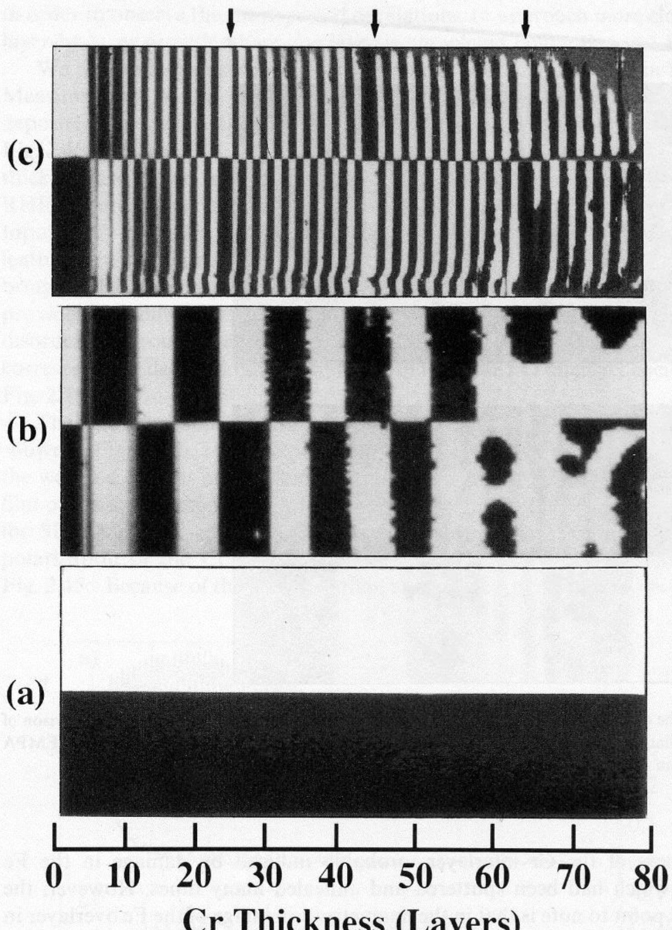


Fig. 2.43. SEMPA image of the magnetization M_y (axes as in Fig. 2.41) showing domains in (**a**) the clean Fe whisker, (**b**) the Fe layer covering the Cr spacer layer evaporated at 30 °C, and (**c**) the Fe layer covering a Cr spacer evaporated on the Fe whisker held at 350 °C. The scale at the bottom shows the increase in the thickness of the Cr wedge in (**b**) and (**c**). The arrows at the top of (**c**) indicate the Cr thicknesses where there are phase slips. The region of the whisker imaged is about 0.5 mm long

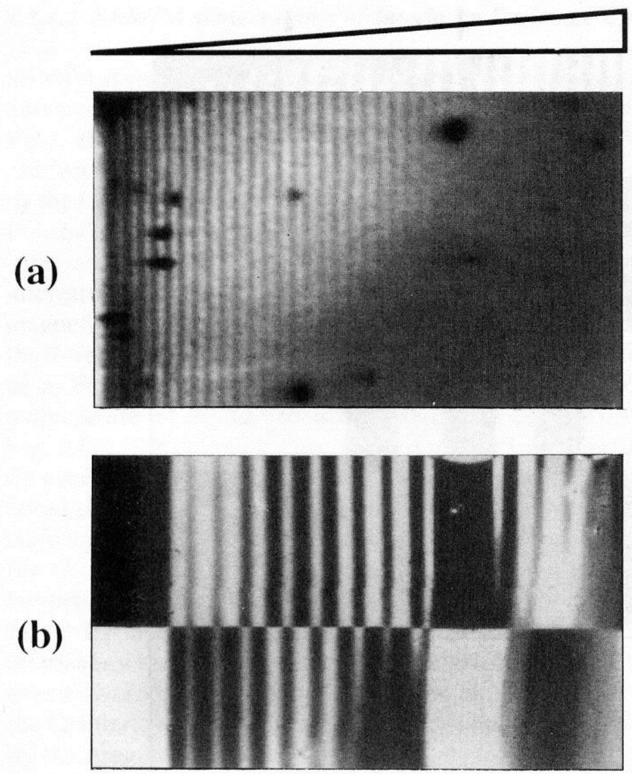


Fig. 2.44. The effect of roughness on the inertlayer exchange coupling is shown by a comparison of (**a**) the oscillations of the RHEED intensity along the bare Cr wedge with (**b**) the SEMPA magnetization image over the same part of the wedge

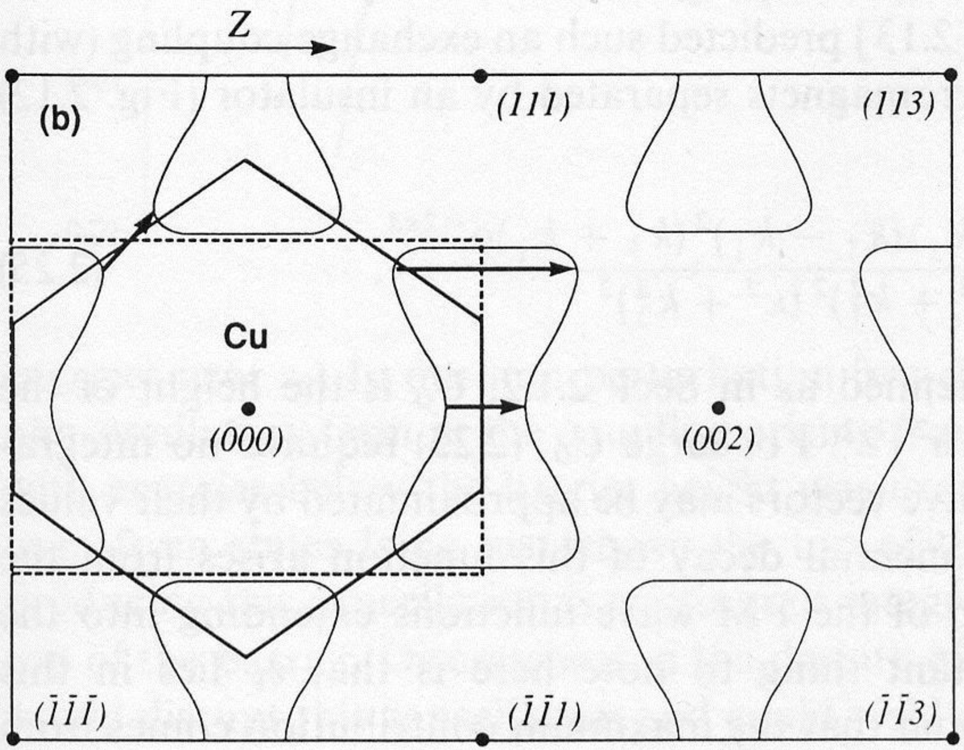


Fig. 2.11. Fermi surface of Cu in the (1 0 0) plane in the extended zone scheme. Arrows indicate values of $2(k_F - G)$ for reciprocal lattice vectors G which can give rise to oscillations with periods greater than π/k_F

Oscillatory magnetic coupling in multilayers

Ru interlayer has the largest coupling strength

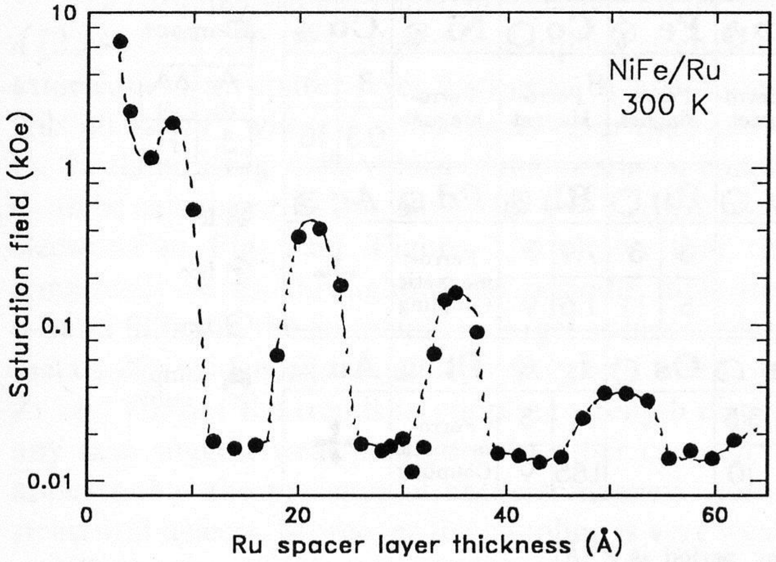


Fig. 2.58. Dependence of saturation field on Ru spacer layer thickness for several series of $Ni_{81}Fe_{19}$/Ru multilayers with structure, 100 Å Ru/[30 Å $Ni_{81}Fe_{19}$/Ru(t_{Ru})]$_{20}$, where the topmost Ru layer thickness is adjusted to be $\simeq$25 Å for all samples

	$\|J_1\|$ at 1st peak (erg/cm^2)	Period (nm)
Cu	0.3	1
V	0.1	0.9
Cr	0.24	1.8
Ir	0.81	0.9
Ru	5.0	1.2

S. S. P. Parkin

Kwo et al, PRB **35** 7295 (1987)
Modulated magnetic properties in synthetic rare-earth Gd-Y superlattices

Spin-dependent conduction in Ferromagnetic metals (Two-current model)

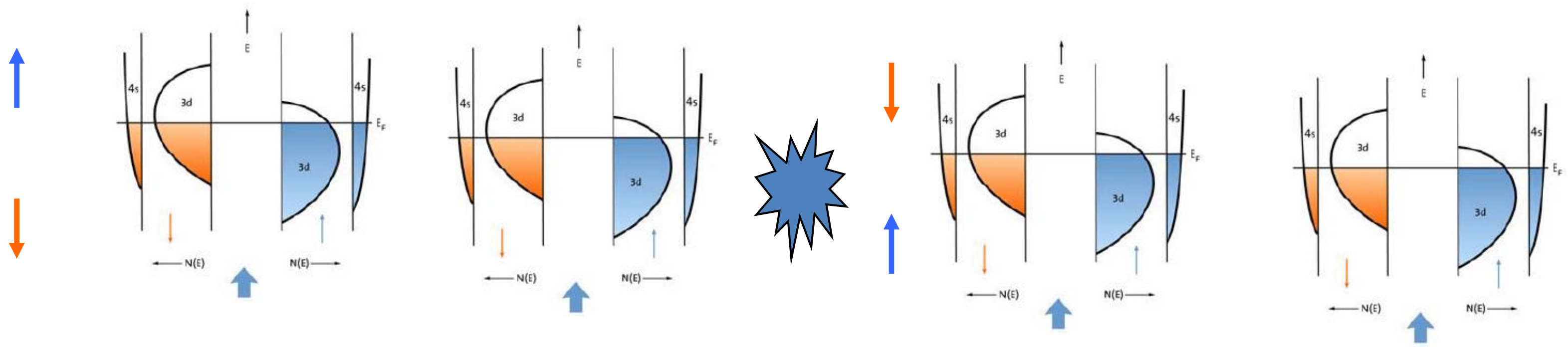
First suggested by Mott (1936)
Experimentally confirmed by I. A. Campbell and A. Fert (~1970)

At low temperature

$$\rho = \frac{\rho_{\uparrow}\rho_{\downarrow}}{\rho_{\uparrow} + \rho_{\downarrow}}$$

At high temperature

$$\rho = \frac{\rho_{\uparrow}\rho_{\downarrow} + \rho_{\uparrow\downarrow}(\rho_{\uparrow} + \rho_{\downarrow})}{\rho_{\uparrow} + \rho_{\downarrow} + 4\rho_{\uparrow\downarrow}}$$



Spin mixing effect equalizes two currents

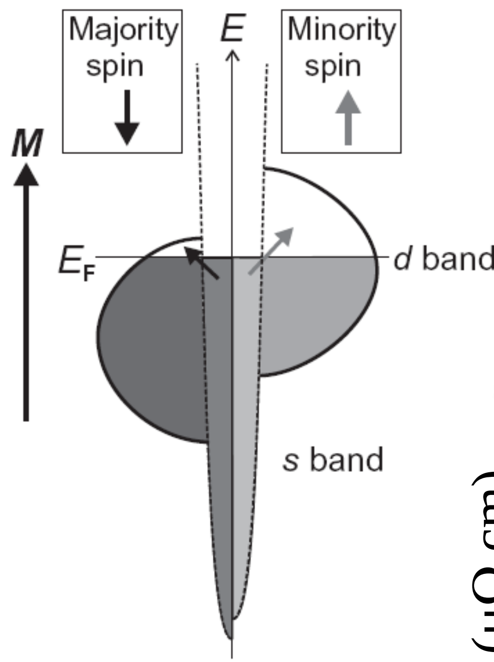
Two Current Model

***s* electrons carry the electric current**

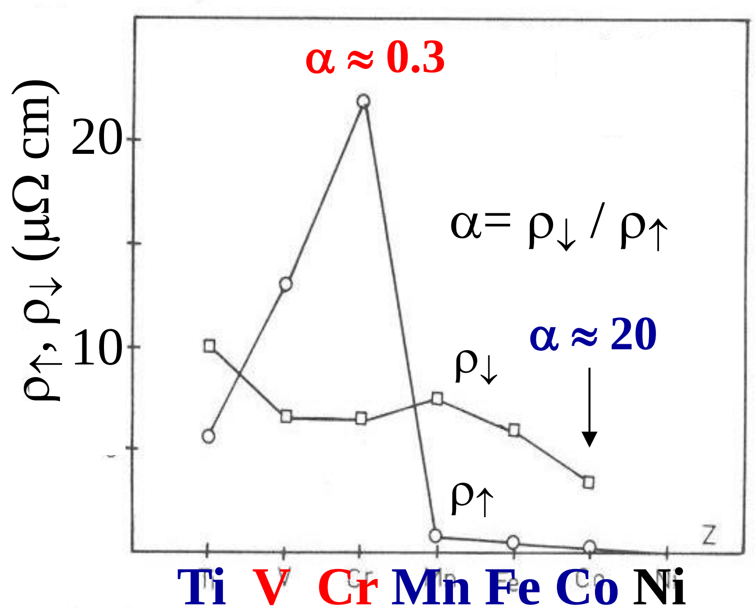
resistivity (spin-dependent *s* → *d* scattering)

$$R^S = \text{const. } N_d^S$$

number of empty *d* states



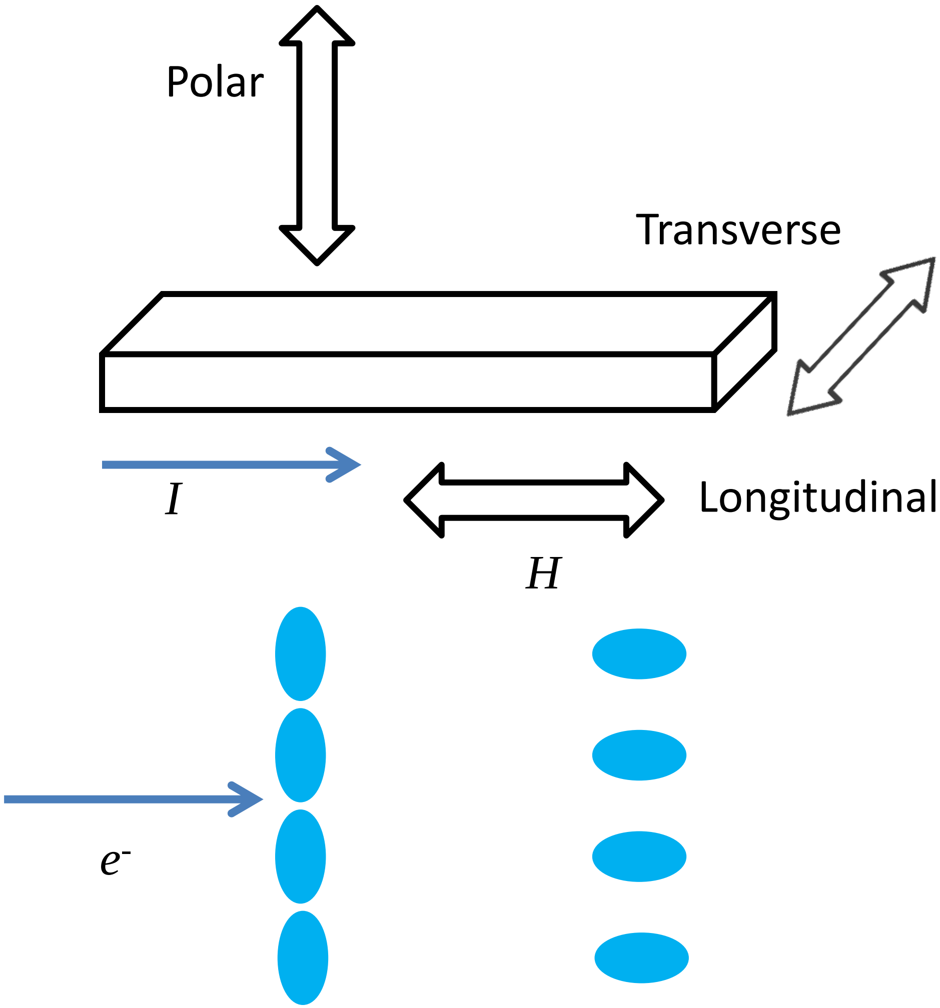
spin selective scattering



element	N_h^d [a]	$\lvert \boldsymbol{m} \rvert$ $[\mu_B]$	R [b] $[\Omega\,\text{m}]$
Fe (bcc)	3.90	2.216	9.71×10^{-8}
Co (hcp)	2.80	1.715	6.25×10^{-8}
Ni (fcc)	1.75	0.616	6.84×10^{-8}
Cu (fcc)	0.50	–	1.68×10^{-8}

A. Fert, I.A. Campbell, PRL **21**, 1190 (1968)

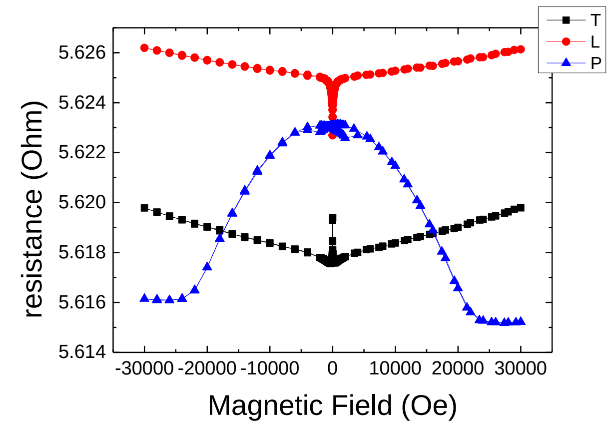
Anisotropic magnetoresistance (AMR)



$$R_{Long.} > R_{Trans.}, R_{Pol.}$$

Geometrical size effect

Fe 100nm 10K



Ni 100nm 10K